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Does Electro-sensing Fish Use the First Order Polarization Tensor for Object Characterization? Object Discrimination Test

(Adakah Ikan yang Mempunyai Elektro-penderiaan Menggunakan Tensor Pengutuban Peringkat Pertama untuk Pencirian Objek? Ujian Pembezaan Objek)

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ABSTRACT

This paper extends the previous works to further explore the role of the first order polarization tensor in electro-sensing by the weakly electric fish specifically for object discrimination and characterization. The first order polarization tensor for few objects used in the considered experiment are calculated and discussed to identify whether there are other evidences to suggest that a weakly electric fish able to recognize the tensor when choosing or rejecting an object. Our findings in this study suggest that all fish during most of the experiments face difficulties to discriminate two objects when their first order polarization tensors are almost similar depending on the types of training given to them.

Keywords: Electrical conducting object; polarization tensor; weakly electric fish

ABSTRAK

Artikel ini membentangkan perbincangan lanjutan daripada kajian lalu untuk mengkaji dengan lebih mendalam peranan tensor pengutuban peringkat pertama elektro-penderiaan oleh ikan deria-elektro lemah dalam pencirian dan pembezaan objek. Tensor pengutuban peringkat pertama bagi beberapa objek yang digunakan dalam eksperimen yang dipertimbangkan telah ditentu dan dibincangkan dalam mencari bukti tambahan untuk menyokong dakwaan bahawa ikan deria-elektro lemah mampu mengenal tensor ini apabila menerima atau menolak sesuatu objek. Hasil kajian mendapati bahawa semua ikan dalam kebanyakan ujian mengalami kesukaran untuk membezakan dua objek apabila tensor pengutuban peringkat pertama bagi kedua-dua objek ini adalah hampir serupa bergantung kepada jenis latihan yang telah diterima oleh ikan-ikan tersebut.

Kata kunci: Ikan deria-elektro lemah; objek mengkonduksikan elektrik; tensor pengutuban

INTRODUCTION

A weakly electric fish is equipped on its body's surface with a single source of electrical voltage and hundreds of voltage sensing cells to perform electro-sensing for navigation, as well as to characterize and locate prey. This kind of fish evolves normally in the rivers of South America or Africa and has a single electric discharge organ which is sometimes also used for communication and in some species as a weapon (Cowan & Fortune 2007; Nelson 2011; von der Emde 2007). A broad spectrum pulse like signal is electrically discharged by the species peter's elephant nose fish *Gnathonemus petersii* while black knife ghost fish *Apteronotus albifrons* generates electric signal closer to a sine wave.

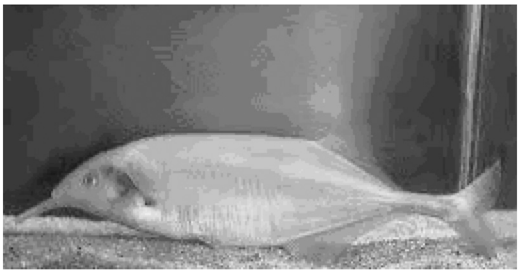

FIGURE 1. A peter's elephant nose fish *Gnathonemus petersii*

The ability of the fish to identify an object through electro-sensing suggests that it performs a complete three dimensional electrical image reconstruction in a real time. On the other hand, the concept of polarization tensor (PT) is studied and explored extensively by Ammari and Kang (2007) not only to describe electrical image but also to improve the reconstruction algorithm of the image. This terminology is applied in many industrial applications such as the electrical impedance tomography (EIT) system, metal detector and material science.

As the movement of the fish through the water before approaching the object acts in a similar way to switch between driven electrodes in an EIT system, it is then meaningful to investigate whether the PT plays some role for object recognition by the fish. This idea is generally motivated with our recent works to use PT in the electrical and electromagnetic applications in describing objects when it is not really necessary to reconstruct the image of the object. Furthermore, fitting electrical image of an object to its PT only offers lower computational cost than fully reconstructing the image.

Therefore, the main purpose of this research is to extend our study in Taufiq and Lionheart (2012) and provide further evidences that support the hypothesis that the first order PT is considered by the weakly electric fish when characterizing objects. During the previous study (Taufiq & Lionheart 2012), we have already investigated the role of the first order PT to the fish during the training conducted by von der Emde and Fetz (2007) where we have found that the fish takes longer time to complete the training to accept and reject two different objects when the first order PT for both objects are almost similar. Now, we will again examine the role of the first order PT to the fish but in another test called as discrimination test which has also been conducted by von der Emde and Fetz (2007).

This article is organized into five sections where it will proceed to the next section with a very brief background about the first order PT. After that, we will summarize the experiment conducted by von der Emde and Fetz (2007) which is considered in this study before displaying and discussing the results of the experiment together with the first order PT. Lastly, this paper ends with few conclusions about the study.

THE FIRST ORDER POLARIZATION TENSOR

The involvement of the first order PT in the mathematical model of electro-sensing fish for object characterization is already explained in Taufiq and Lionheart (2012). In order to summarize it, we first consider $u(x)$ as the electrical voltage generated by the fish, F at any point x in $\mathbb{R}^3$. By assuming $H(x)$ to be the voltage without any object in the region exterior to the fish, $\mathbb{R}^3 - F$, the perturbated voltage $u(x) - H(x)$ due to an inclusion B in the same region can be mathematically expressed under certain conditions as

$$(u - H)(x) = -\nabla\Gamma(x)M\nabla H(0) + O(1/|x|^{-2}), \tag{1}$$

where the origin $O \in B$, $\Gamma(x) = -(4\pi|x|)^{-1}$ and M is the first order PT. Thus, instead of analyzing the full (1), one can simply refer to M to understand B. The first order PT, M in this case can be determined by solving an integral operator in the following definition as suggested by Ammari and Kang (2007).

Definition 1 (The first order PT of an object B) The first order polarization for a Lipschitz bounded domain $B \in \mathbb{R}^3$ containing the origin O at conductivity $0 < k \neq 1 < +\infty$ denoted by $M(k, B)$ is a 3×3 matrix in the form

$$M(k,B) = \begin{Bmatrix} M_{(1,0,0)(1,0,0)} & M_{(1,0,0)(0,1,0)} & M_{(1,0,0)(0,0,1)} \\ M_{(0,1,0)(1,0,0)} & M_{(0,1,0)(0,1,0)} & M_{(0,1,0)(0,0,1)} \\ M_{(0,0,1)(1,0,0)} & M_{(0,0,1)(0,1,0)} & M_{(0,0,1)(0,0,1)} \end{Bmatrix}, \tag{2}$$

where

$$M_{ij}(k,B) = \int_{\partial B} y^j \phi_i(y)\, d\sigma(y). \tag{3}$$

For $y \in \partial B$, ∂B is the boundary of B and both i,j are multi indices. Here, $\phi_i(y) = (\lambda I - K_B^*)^{-1}(vx \cdot \nabla x^i)$ for $x \in \partial B$, I is the appropriate identity matrix and v_x is the outer unit normal vector to the boundary ∂B at the point x. K_B^* is also an integral operator over the boundary of B defined with the Cauchy principal value P.V by:

$$K_B^*\phi(x) = \frac{1}{4\pi}\,\mathrm{p.v.}\int_{\partial B} \frac{(x-y)\cdot v_x}{|x-y|^3}\phi(y)\, d\sigma(y), \tag{4}$$

where $|x - y|$ is the distance between x and y.

Definition 1 suggests that the first order PT depends only on B and its conductivity and thus can be directly calculated if the object B is known. Fortunately, von der Emde and Fetz (2007) have conducted several experiments and suggested that *petersii* elephant nose fish can recognize several parameters of two different objects and hence distinguish them. Therefore, in this study, we will calculate the first order PT for objects used in one of the experiment by the method proposed by Taufiq and Lionheart (2013) to investigate whether the fish also recognize the first order PT when characterizing objects.

EXPERIMENTAL DETAILS

In their study, von der Emde and Fetz (2007) conducted their experiments to eight *Gnathonemus petersiis* Gunther 1832 in order to verify whether all fish were able to detect and distinguish objects in the water through electrolocation. However, this research will investigate the PT for only 5 fish from their experiment which are Fish 1, 2, 3, 7 and 8 where they are renamed here as Fish B, A, E, C and D, respectively. Now, we will discuss briefly about one of their experiments which are considered in this study which is called as the 'discrimination test' (DT).

Before DT was conducted to each fish, all fish were trained first to accept and reject two different objects with rewards until they were able to choose the correct object at 75% level of training for 3 consecutive days. Some control and test experiments were also performed after that to ensure that those fish depended only on their electrical sense while making decision in choosing the correct object. The DT was then executed when the authors (von der Emde & Fetz 2007) were sure that the fish was familiar to what it should accept and reject at more than 75% level of testing after some period from the training.

During DT, one of the trained objects (either the acceptance or the rejection object) was replaced by another different object with similar material and height. In order to clarify whether the training influenced the fish in choosing the particular object, all fish now were asked to discriminate the pair of new object and either acceptance or rejection object with no reward or punish anymore. At this moment, the percentages of still; choosing the original acceptance object when the rejection object was replaced with the new object; and accepting the new object when the original acceptance object was replaced were recorded for analysis.

In order to investigate the role of the first order PT in the DT, the first order PT for few objects used in the test conducted to the fish by von der Emde and Fetz (2007) will be firstly calculated. We will then examine and analyze these values together with the DT to see whether the first order PT influences the fish when approaching or rejecting certain objects. This will be discussed in the next section.

RESULTS AND DISCUSSION

This section begins with the presentation of the characteristic of the first order PT for several considered objects used during the training and the DT in von der Emde and Fetz (2007). The characteristics between the two pair objects in each experiment will be compared and displayed in few tables after this. These values are then discussed to justify the role of the first order PT in the DT.

The First Order PT for Several Objects Based on the objects listed in Table 1 and their dimension, the first order PT for each object was calculated according to the numerical method in Taufiq and Lionheart (2013). Since the objects used in von der Emde and Fetz (2007) are made from highly electrical conductive metal, the first order PT for all of them in this study were calculated at conductivity 10^5. Similarly to Taufiq and Lionheart (2012), instead of considering all 9 numbers of M and also to reduce numerical errors in computation, the eigenvalues of M were calculated and shown in the same table to characterize M as suggested by Ammari and Kang (2007) and Anton and Rorres (2000).

In this study, the eigenvalues of each first order PT are numerically determined to two significant figures. Based on symmetrical property of the object and the theory in Ammari and Kang (2007) and Danielson (1997), these objects must have at most two distinct eigenvalues such that there are two eigenvalues that are almost equal for cone, cylinder and pyramid which suggest they are actually repeated while cube and sphere has only one eigenvalue. Due to numerical errors arise from the approximated integration, eigenvalues that are close are replaced in Table 1 by their average. The ratio of the non repeated to the repeated eigenvalues of each first order PT for the object is then defined as $\hat{c}$ and determined. This $\hat{c}$ will be used to describe and measure the first order PT for each object similarly as in Taufiq and Lionheart (2012).

Relating the First Order PT to the DT Our efforts in this section is to explain the DT conducted by von der Emde and Fetz (2007) for fish A, B, C, D and E as well as relating their findings to the first order PT in order to investigate the role of the first order PT in the test. The results from von der Emde and Fetz (2007) about the DT will be first summarized and displayed where the percentages of still choosing and rejecting original object (or accepting new object) are combined in one table. It can be seen later on from the tables that the same new objects, N are used to replace either original trained acceptance (+) or rejection (-) object of each test for all fish.

The table will also include the absolute value of the difference between $\hat{c}$ of the pair tested objects (+ and N or N and -) denoted as v_{DT} which also represent the difference of the first order PT between the pair. In this case, the pair of objects is becoming similar to each other electrically when the value v_{DT} between the objects is getting smaller. The results linking the DT and the first order PT for different fish which are displayed in the tables will be discussed now as follows.

Fish A and B Table 2 shows the result of DT for Fish A and B where both of them are originally trained to accept pyramid and reject cube. Based on this table, the percentage for Fish A and B to still choose pyramid (+) when the new object N replaces cube (-) increase as the value $v_{DT(N\,\&+)}$ increase. This suggests that the difference of the first order PT between pair tested objects influence both fish in making decision where both fish are claimed to accept pyramid (+) as they are trained before at lower percentage when pyramid (+) is almost similar to N i.e v_{DT} between pyramid (+) and N is smaller.

In contrast, when pyramid (+) is replaced by N, the percentages to choose N or to reject cube (-) looks high during each test for whatever difference of the first order PT between the pairs. Obviously, since the difference of the first order PT between N and pyramid (+) when N is cone is smaller than the difference between cone and cube (-), both fish are suggested to easily accept cone which

TABLE 1. The first order PT for several objects

Object	Dimension (cm)	Eigenvalues	$\hat{c}$
Cone	3(d), 3(h)	30, 30, 25	0.83
Cube	3×3×3 (l×w×h)	97, 97, 97	1.00
Cylinder	3(d), 3(h)	66, 66, 80	1.21
Pyramid	3×3×3 (l×w×h)	44, 44, 32	0.73
Sphere	3(d)	42, 42, 42	1.00

d = diameter,
l = length, w = width, h = height

is similar to pyramid (+) since they are trained to accept pyramid (+). However, if the difference of the first order PT between *N* and cube (-) is smaller than the difference between *N* and + such as when *N* is sphere or cylinder then the percentage of accepting *N* is expected not to be too high due to *N* is more similar to cube (-) than pyramid (+) which means that both fish accept *N* at high percentage probably because they are trained to reject cube (-).

Fish C and D The performance of Fish C and D based on Table 3 can also be explained in a similar way such that both fish choose cone (+) when cube (-) is replaced by pyramid at around 63% in average and the percentage jump to 95% when cone (+) is replaced by pyramid as both of them are trained to accept cone (+) and it can be seen that the difference of the first order PT between cone (+) and pyramid (*N*) is smaller if compare between pyramid (*N*) and cube (-). When cone (+) or cube (-) is replaced by sphere, the performances of both fish are different since they accept cone (+) at around 63% in average and reject cube (-) at 95% although the difference of the first order PT between sphere (*N*) and cube (-) is less than the difference of the first order PT between sphere (*N*) and cone (+). The higher percentage of accepting sphere in this case suggests that both fish reject cube (-) strictly because they are trained to reject cube (-).

Fish E According to Table 4, the value of v_{DT} between the original cone (+) and each new object *N* for Fish E can be clearly seen to be smaller than v_{DT} between each *N* and pyramid (-). Therefore, if the performance of the fish in discriminating between each *N* and either original cone (+) or pyramid (-) depends only on the difference of the first order PT between pair tested objects, it is expected that the percentage of choosing original cone (+) for each test pair cone (+) and *N* is small due to small difference of the first order PT between cone (+) and each *N*. However, the percentage of still choosing the original cone (+) here is very high possibly caused by the training.

Conclusion

This study showed that the fish during the experiment face difficulties to accept or reject two different objects when the difference of the first order PT between these objects is small. In other words, fish might electrically recognize two objects as the same when their first order PT are similar. Thus, these evidences suggest that the first order PT plays some part during DT and hence electro-sensing by the fish as claimed in the previous study. Furthermore, these findings of course depend on the training given to the fish which agree with the previous researchers that the training also influences the fish in recognizing object. Perhaps, other

TABLE 2. DT for Fish A and B

N	Preserve +Accept + (%)		$v_{DT(+\&N)}$	Preserve -Accept *N* (%)		$v_{DT(N\&-)}$
	Fish A	Fish B		Fish A	Fish B	
Cone	55	45	0.10	95	95	0.17
Sphere	60	55	0.27	90	90	0.00
Cylinder	80	80	0.48	80	70	0.21

+ : pyramid
- : cube

TABLE 3. DT for Fish C and D

N	Preserve + Accept + (%)		$v_{DT(+\&N)}$	Preserve Accept *N* (%)-		$v_{DT(N\&-)}$
	Fish C	Fish D		Fish C	Fish D	
Pyramid	60	65	0.10	95	95	0.27
Sphere	65	60	0.17	95	95	0.00

+ : cone
- : cube

TABLE 4. DT for Fish E

N	Preserve + Accept + (%)	$v_{DT(+\&N)}$	Preserve -Accept *N* (%)	$v_{DT(N\&-)}$
Cube	95	0.17	60	0.27
Sphere	90	0.17	50	0.27
Cylinder	95	0.38	50	0.48

+ : cone
- : pyramid

experiment about object recognition by electro-sensing fish can also be investigated to further justify the role of the first order PT in it.

As the first order PT is one of the parameter of the object which can only be electrically recognized, these study also highlights it's sensed and fitted by the fish to electrically describe object. This amazingly happens in the behavior of one of the living creatures in the world that has been provided with only small brain. Therefore, similar concept can be learned and adapted by us to apply the PT in any real related physical applications in the future.

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